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Bayesian Decisions with Ambiguous Belief Aversion

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Abstract

This study provides an empirical perspective on the effect of ambiguous environmental risk information on lottery preferences using a sample of 646 adults. The learning process follows a Bayesian expected utility model in terms of the overall magnitude and sign of the weights that respondents place on the risk information. Significant ambiguous belief aversion that is consistent with the Ellsberg paradox is also evident. The extent of this aversion increases with the size of the risk spread, but at a decreasing rate. These results are consistent with both probability-based and preference-based models of ambiguous probabilities. The findings also indicate the presence of cognitive limitations in the processing of risk information, but lead to rejection of more extreme models in which individuals respond in alarmist fashion or do not learn at all.

A substantial economic literature has documented several manifestations of irrationality in decisions under uncertainty.[1] One of the most long-standing and prominent anomalies in the choice under uncertainty literature has been the Ellsberg (1961) paradox, which focuses on the influence of ambiguous probabilistic information.

Consider a situation in which you will win a prize if you can correctly guess the color of a ball that will be drawn from an urn. Urn 1 contains 50 red balls and 50 black balls. Urn 2 contains an unknown mixture of red and black balls, but you have the option of selecting the color of the ball that may be drawn. Subjects generally prefer picking a ball from urn 1 with the hard probabilities to drawing from the urn with an uncertain mixture. As Raiffa (1961) observed, this preference for precise probabilities is not rational, since an individual could convert the "soft" probability for urn 2 into a hard probability by flipping a fair coin and relying on the outcome of this coin toss to select the color of the ball to be drawn. Notwithstanding the irrationality of this aversion to soft probabilities of winning a prize for one-shot lotteries such as this, this behavior has been borne out in a number of other studies of individual attitudes toward ambiguous risks.[2]

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The Ellsberg paradox indicates that in situations of winning a *prize*, individuals would prefer a hard probability of success to an equivalent soft probability; however, in situations in which individuals will incur a *loss* rather than experience a gain, would individuals prefer a sure probability of a loss or a less precisely understood probability of equivalent magnitude? Evidence presented in the insurance context by Kunreuther and Hogarth (1989, 1990) suggests that there is aversion to ambiguous probabilities in the case of losses, but the insurance-experiment results in Camerer and Kunreuther (1989b) are more mixed. The character of this risk ambiguity may also be of consequence. Heath and Tversky (1991), for example, found that ambiguity was of particular concern when decision-maker competence was an issue.

The influence of risk ambiguity in ways not predicted by expected utility theory has led to the development of alternative models of choice under uncertainty. In some instances these alternative theories relax the additivity assumption for probabilities;[3] in their analyses the authors speculate that the participant in the study believes that the experiment is being manipulated against him in the case of the urn with uncertain properties;[4] and a final class of models hypothesizes that the probabilities are additive but that there is an additional component of a multiattribute utility function pertaining to ambiguity, such as regret or blame associated with ambiguous choices.[5]

A variety of frameworks have also been suggested to capture the empirical aspects of this behavior. Einhorn and Hogarth (1985, 1986) have explored attitudes toward ambiguous risks using an anchoring and adjustment model whereby individuals initially set their probability at some anchor value, and then alter this probability depending on the information that has been received. Although this response to ambiguous probabilities potentially does violate standard expected utility models, the anchoring and adjustment process is very similar in character to a Bayesian learning procedure. Einhorn and Hogarth's anchor and adjustment model hypothesizes that individuals' assessed probabilities are nonadditive, however, which contradicts the Bayesian approach. The studies by Kunreuther and Hogarth (1989, 1990) examine this formulation within a series of experiments pertaining to insurance. Kunreuther and Hogarth's (1990) experimental study found that insurers added an ambiguity premium when setting insurance rates for hypothetical ambiguous risks. This behavior does not necessarily contradict the Bayesian expected utility model, since risk ambiguity may affect portfolio variance when there are multiple risks.[6] Heath and Tversky (1991) likewise find that risk ambiguity is of consequence, but the role of individual knowledge of the lottery context leads them to dismiss not only Bayesian models, but post-Bayesian subadditive probability models as well.

The most distinctive feature of our study is that the empirical reference point that will serve as the basis of the hypothesis tests will be a model of expected utility that incorporates a Bayesian learning process. The character of the risk information provided will also take into account the multiplicity of informational sources that may influence individual's probabilistic beliefs.

The nature of risk ambiguity that will be of particular concern here is the effect of receiving conflicting environmental risk information. Situations in which there is ambiguous risk information occur frequently. Scientific and technical studies often have different implications for the magnitude of a risk. How do individuals process this risk information and act upon it when making their decisions, and can this behavior be reconciled with standard models of expected utility?

Section 2 of this article outlines the model of decision that will be tested, and section 3 discusses the estimating equation. Section 4 presents the empirical results. Risk information does affect decisions in a manner that a Bayesian decision model would predict. However, there are additional influences that cannot be explained using a rational learning approach, including both a role of ambiguous belief aversion as well as cognitive limitations in processing risk information.

1. The lottery structure and implications for expected utility theory

1.1. The survey approach

In the case of the Ellsberg model, the uncertain decision context involved an unspecified mixture of balls in a Bernoulli urn. Our focus will be somewhat different. In particular, the ambiguity in the risk information arises from conflicting scientific information that the individual has received about the health risk associated with an environmental pollutant. Scientific evidence is especially imprecise for dimly understood health risks. If individuals receive different pieces of risk information, how do they process this information in forming their risk judgments and in making their decisions? Moreover, is there evidence of ambiguous belief aversion? If, for example, individuals were presented with two studies indicating annual risks of 140×10^{-6} and 160×10^{-6}, would this information be viewed as more or less favorable than two studies indicating risks of 100×10^{-6} and 200×10^{-6}? The mean risk level is the same, but the spread between the two risks is greater for the second pair of studies. If the assessed risk is higher in the latter case, individuals are said to exhibit ambiguous belief aversion. In testing for the presence of ambiguous belief aversion, the analysis below will take into account the different weights that individuals may place on risk information depending on the order of presentation of multiple risk estimates as well as whether the results of the studies are communicated simultaneously or sequentially.

Because of the substantial length of the interactive program discussed below, we will only summarize the general survey approach.[7] Individuals participating in this study received a computer-administered survey that addressed their willingness to move to different areas that differed in terms of the risks they posed. More specifically, individuals had a choice of moving to area A or area B, which differed in terms of their environmental risks. Part of the sample encountered a risk of a nonfatal nerve disease, and in other cases the risk context was lymphoma, a cancer of the lymph system. In each case, the risk was specifically linked to environmental pollution. The survey informed the subjects that the areas were otherwise identical to the areas in which they now lived. Moreover, subjects were told that the risk levels were less than in their present location, thus avoiding possibly alarmist responses to increases in the risks that have been observed in some studies. After receiving a short description of the diseases, the subjects answered a series of questions pertaining to each of the consequences of the disease that was intended to reinforce their understanding of the impact of the ailments on their health and activities.

The interactive computer program then informed individuals of the risk in terms of the total number of cases of the disease per one million population that was implied by each of the studies for area A. They were then asked what precise risk in area B would be equivalent to the risks posed in area A, for which they had received two pieces of risk information. The equivalent risk in area B was ascertained through a series of iterative paired comparisons that were modified until indifference was reached. The survey established this equivalence by, in effect, determining the hypothetical reference lottery in area B that the respondent viewed as being indifferent to the lottery with ambiguous risks in area A.

1.2. The lottery structure of the survey

To make the survey procedure more explicit, we will utilize the following notation. Let $U(Y)$ be the utility of good health with income Y, and $V(Y)$ be the utility of ill health with income Y, where $U(Y) > V(Y)$. The respondent receives information about two studies pertaining to the risk in area A, where each study i has associated illness frequency rate r_i and informational context ξ_i, where $i = 1, 2$. The risk level r_1 is provided to the respondent, and the informational context ξ_i is a parameter that must be estimated based on the implicit weight the subject places on the risk information. Thus, the respondent treats study i as equivalent to observing ξ_i trials in which the disease may occur, where the disease occurs in a fraction r_i of these trials. After receiving this risk information, the respondent forms an assessed probability of illness $p(r_1, \xi_1, r_2, \xi_2)$ in area A.

The objective of the survey is to ascertain the precisely understood probability of illness s that establishes indifference between area A and area B. The value of s satisfies

$$p(r_1, \xi_1, r_2, \xi_2)V(Y) + (1 - p(r_1, \xi_1, r_2, \xi_2))U(Y) = sV(Y) + (1 - s)U(Y), \quad (1)$$

or, upon simplification,

$$s = p(r_1, \xi_1, r_2, \xi_2). \quad (2)$$

The function of the reference lottery is to establish the assessed probability of illness that the respondent attributes to the risk after being given the ambiguous risk information.

1.3. Ambiguous belief aversion

Consider two ways in which ambiguous belief aversion could enter the respondent's evaluation of the reference lottery risk s that establishes equivalence with the ambiguous lottery. First, ambiguous beliefs may affect the assessed probabilities of the outcomes. The Bayesian formulation of $p(r_1, \xi_1, r_2, \xi_2)$ explicitly recognizes that ambiguous risk information will affect risk assessments. Problems arise, however, if the assessed probabilities of the two outcomes are not additive. Suppose that in the context of ambiguous beliefs the individual adjusts downward his or her assessed probability of the favorable outcome (in our example, good health) but does not alter the assessed probability of the adverse outcome.[8] This formulation will generate aversion to ambiguous choices of winning a prize in the Ellsberg experiment and reluctance to incur more ambiguous chance of a loss.

To demonstrate this conclusion in the case of a loss, let $a(r_1, \xi_1, r_2, \xi_2)$ be the ambiguity belief aversion value that affects the assessed risks.[9] The reference lottery that is equivalent to the ambiguous risk lottery is defined by

$$sV(Y) + (1 - s)U(Y) = p(r_1, \xi_1, r_2, \xi_2)V(Y) + (1 - p(r_1, \xi_1, r_2, \xi_2) - a(r_1, \xi_1, r_2, \xi_2))U(Y). \quad (3)$$

Solving for s yields

$$s = p(r_1, \xi_1, r_2, \xi_2) - \frac{a(r_1, \xi_1, r_2, \xi_2)U(Y)}{V(Y) - U(Y)}. \quad (4)$$

If ambiguous belief aversion enters through the probabilities, then the reference lottery value s differs from the Bayesian risk assessment $p(r_1, \xi_1, r_2, \xi_2)$ by the final term, which is dependent on the utility of the two states and the effect of ambiguous beliefs on the assessed probability of the favorable event. By assumption, the ambiguity belief aversion term $a(r_1, \xi_1, r_2, \xi_2)$ is positive, implying that the assessed risk s exceeds the Bayesian risk value $p(r_1, \xi_1, r_2, \xi_2)$. A similar demonstration indicates that there is an aversion to ambiguous choices of winning a prize as compared with a precise probability.

Alternatively, let ambiguity enter the model not through the probabilities, but through the utility function. If the adverse outcome occurs in the case of ambiguous beliefs, let there be some regret or blame associated with the event.[10] If respondents have a multi-attribute utility function for which risk ambiguity is an additively separable component $A(r_1, \xi_1, r_2, \xi_2)$ that reduces the value of the adverse event, then the reference lottery satisfies

$$sV(Y) + (1 - s)U(Y) = p(r_1, \xi_1, r_2, \xi_2)(V(Y) - A(r_1, \xi_1, r_2, \xi_2)) + (1 - p(r_1, \xi_1, r_2, \xi_2))U(Y). \quad (5)$$

Solving for s, we obtain

$$s = p(r_1, \xi_1, r_2, \xi_2) - \frac{p(r_1, \xi_1, r_2, \xi_2)A(r_1, \xi_1, r_2, \xi_2)}{V(Y) - U(Y)} \quad (6)$$

Again, ambiguity creates an aversion to incurring a loss and decreases the attractiveness of potentially winning a prize.

Both the probability-based and utility-based models of ambiguous belief yield formulations in which the equivalent reference lottery probability s equals the Bayesian probability

$p(r_1, \xi_1, r_2, \xi_2)$ minus a complex ambiguous belief aversion term. It is useful to have a measure of ambiguous belief aversion that is independent of the particular formulation for capturing the role of ambiguous risk information. We will define the degree of ambiguous belief aversion (DABA) to be the difference between the equilibrating probability s in the reference lottery and the Bayesian probability assessment, or

$$\text{DABA} = s - p(r_1, \xi_1, r_2, \xi_2). \tag{7}$$

The magnitude of the ambiguous belief aversion term DABA is given by $a(r_1, \xi_1, r_2, \xi_2)U(Y)/(U(Y) - V(Y))$ for probability-based models and by $p(r_1, \xi_1, r_2, \xi_2)A(r_1, \xi_1, r_2, \xi_2)/(U(Y) - V(Y))$ for utility-baed models.

The effect of the ambiguous belief aversion term on the equilibrating probability s depends on the lottery structure and the value of the payoffs. Increases in the value of the utility of the preferred state of nature $U(Y)$, for any given value of $V(Y)$, will decrease the DABA value for each of the two models.[11] Similarly, higher values of $V(Y)$ will increase the value of DABA. The role of ambiguous belief aversion will consequently depend both on the probabilistic structure of the lotteries as well as on the utility of the lottery payoffs. Results that have indicated the dependence of the influence of risk ambiguity on the lottery consequences, such as those of Kunreuther and Hogarth (1989) and Heath and Tversky (1991), consequently are consistent with each of these formulations.[12]

2. The empirical framework

2.1. The equation to be estimated

Because of the potentially complex functional form of the DABA term that affects s, the empirical analysis will estimate an average value of ambiguous belief aversion to determine whether there is any significant discrepancy between s and $p(r_1, \xi_1, r_2, \xi_2)$. Evidence that the equilibrating probability s exceeds the Bayesian probability $p(r_1, \xi_1, r_2, \xi_2)$ will lead to a rejection of the conventional Bayesian expected utility model, but will not indicate whether the probability-based model or a preference-based model of ambiguous belief aversion has greater validity.

Suppose that respondents have the unobserved prior risk assessments of disease equal to p_0, with associated precision γ(i.e., respondents act as if they have observed γ trials in forming their prior, where a fraction of p_0 of the trials involve occurrence of the disease).[13] For a Bayesian learning model with probability assessments that can be characterized by a beta distribution,[14] the posterior assessed probability of a disease p_1 after learning of study 1 is given by

$$p_1 = \frac{\gamma p_0 + \xi_1 r_1}{\gamma + \xi_1}. \tag{8}$$

After receiving information pertaining to study 2, the revised posterior probability assessment becomes

$$p_2 = \frac{(\gamma + \xi_1)p_1 + \xi_2 r_2}{\gamma + \xi_1 + \xi_2} = \frac{\gamma p_0 + \xi_1 r_1 + \xi_2 r_2}{\gamma + \xi_1 + \xi_2}. \quad (9)$$

If we let $\gamma' = \gamma/(\gamma + \xi_1 + \xi_2)$, $\xi_1' = \xi_1/(\gamma + \xi_1 + \xi_2)$, and $\xi_2' = \xi_2/(\gamma + \xi_1 + \xi_2)$, then equation (9) can be expressed in terms of a simple weighted average of the prior probability, the risk implied by study 1, and the risk implied by study 2. The weights are the relative informational content associated with each of these components. Thus

$$p_2 = \gamma' p_0 + \xi_1' r_1 + \xi_2' r_2. \quad (10)$$

It should be noted that the estimate of $\gamma' p_0$ will reflect the relative informational content and level of respondents' prior beliefs. The estimates of ξ_1' and ξ_2' capture the informational content placed on the risk information received and will explicitly take into account the role of respondent knowledge in processing risk information. Differences in individual knowledge have a fundamental role to play within a Bayesian learning model, and the estimates of equation (10) will explicitly recognize this dependence. Heath and Tversky (1991) found that individual competence also influences the role of risk ambiguity. The estimates derived from equation (10) will distinguish the role of risk ambiguity from whatever legitimate influence risk competence plays within Bayesian learning models. Unlike Heath and Tversky (1991), however, we do not also include experimental treatments for which there will be variations in risk competence that influence the role of ambiguity.

2.2. *Empirical hypotheses*

Several learning situations that are summarized in Table 1 can be distinguished. The first is what will be designated as the "naive Bayesian." Respondents may learn (i.e., $\xi_1', \xi_2' > 0$), and in doing so they place an equal weight on the two studies regarding the area A risk (i.e., $\xi_1' = \xi_2'$). For experimental treatments that treat the studies symmetrically, this response is reasonable.

Table 1. Hypotheses for alternative learning models

Nature of Learning	Empirical Hypotheses
Naive Bayesian	$\xi_1' = \xi_2' > 0$; $1 \geq \xi_1' + \xi_2'$; $\gamma' p_0 \geq 0$.
Attentive Bayesian	$\xi_2' > \xi_1' > 0$ if temporal order; $\xi_1' = \xi_2' > 0$ if no temporal order; $1 \geq \xi_1' + \xi_2'$; $\gamma' p_0 \geq 0$.
Bayesian with cognitive limitations	$\xi_1' > \xi_2'$ or $\xi_2' > \xi_1'$ if no temporal order; $\gamma' p_0 \geq 0$.
Alarmist learner	$\xi_1' > 1$, $\xi_2' > 1$, or $\xi_1' + \xi_2' > 1$, or $\gamma' p_0 + \xi_1' + \xi_2' > 1$.
Ambiguous belief aversion	$\Psi_1 R + \Psi_2^2 R^2 > 0$; $\Psi_1 > 0$ if quadratic term excluded.

Some of our experimental treatments attribute an explicit temporal order to the presentation of the results of the different scientific studies. In situations in which the second study is undertaken after study 1, one might reasonably conclude that study 2 has greater scientific validity, since it presumably extends study 1. Respondents whom we will designate as "attentive Bayesians" consequently should place a greater weight on the second study if there is an explicit temporal order. In these cases, $\xi_2' > \xi_1' > 0$.

A third possibility is that of a Bayesian with cognitive limitations. In situations in which information about two studies is acquired and there is no temporal order, the studies should be viewed symmetrically. If this information were provided over a period of time, one would expect individuals to place greater weight on the second study because of a recency effect. However, if the information is presented simultaneously on a computer screen (as in this study), then there should be no recency effect arising from temporal differences in information acquisition. Respondents may, however, place a greater weight on the first study if they are not attentive to the survey task, or they may weight the second study more highly if they infer a temporal order when none existed. Thus, $\xi_1' \neq \xi_2'$, but the direction of the discrepancy depends on the character of the respondents' cognitive errors.

In each of these instances, the learning process will not satisfy the Bayesian updating process if the response to the information provided is too great. In particular, the sum of the relative informational weights must equal 1 (i.e., $\gamma' + \xi_1' + \xi_2' = 1$). In the analysis below, we will be unable to estimate γ', since only $\gamma' p_0$ can be estimated. Thus, the test for an "alarmist learner" is whether the relative informational weights on the two studies exceed 1 either individually or collectively (i.e., $\xi_1' > 1, \xi_2' > 1, \xi_1' + \xi_2' > 1$, or $\gamma' p_0 + \xi_1' + \xi_2' > 1$). Responses of this type will suggest that individuals overreact to risk information that they receive. Many observers have noted that there are often alarmist responses with respect to publicly identified low-probability events, such as the chance of being killed in a terrorist attack while vacationing in Europe or the risk of being poisoned by a Chilean grape tainted with cyanide. Whether such extreme responses contradict a Bayesian learning model[15] can be tested by assessing whether the learning equation could potentially lead to assessed probabilities in excess of 1.0.

These four learning models capture different variants of learning behavior that reflect modifications that are consistent with the learning model formulated in equation (10) or involve alternative hypotheses regarding the magnitude and signs of the coefficients. Another class of models pertains to the role of ambiguous belief aversion. For the probability-based ambiguity model (see equation (4)) and the preference-based ambiguity model (see equation (6)), the equilibrating value of s will differ from $p(r_1, \xi_1, r_2, \xi_2)$ by a complex ambiguous belief term. The equations to be estimated will incorporate a risk ambiguity aversion variable to test for this influence.

In situations in which there is a broad range of scientific evidence regarding the risk, the respondent faces a less precisely understood risk. We will model the role of risk ambiguity through inclusion of a variable equal to the risk range R implied by the studies, where $R = |r_1 - r_2|$. The R term captures one of the most salient aspects of risk ambiguity and will enable us to estimate the average value of risk ambiguity aversion for the sample. The role of some of the other factors that might affect ambiguous belief aversion will be explored through the use of interaction terms with the risk range R.

To better assess the empirical consequences of this formulation, consider two sets of information. For the information set I, the two studies indicate annual risks of 100×10^{-6} and 200×10^{-6}, implying that the value of R is 100×10^{-6}. Information set II's studies indicate risks of 125×10^{-6} and 175×10^{-6}, with a risk range of 50×10^{-6}. In each case the mean risk is 150×10^{-6}, but the value of R is greater for the more ambiguous study pair.

A difference in the assessed value of the risk s that is indifferent to the risk implied by the sets of studies does not necessarily imply that respondents exhibit ambiguous belief aversion. For the two sets of risk information specified above, there would be no ambiguous belief aversion if the assessed risk s were 150×10^{-6}. Suppose, however, that respondents assess the value of s associated with information set I as equalling 166.7×10^{-6}, and the assessed value of s for information set II is 158.0×10^{-6}. Both pairs of studies had the same median risk, and information set I had a risk range R of 100×10^{-6}, as compared with only 50×10^{-6} for set II. The assessed value of s is greater for the pair of studies with a greater risk range. One might conclude from these results that subjects exhibit ambiguous belief aversion in all cases involving imprecisely understood probabilities that the extent of aversion increases with the size of the risk range R. This conclusion may be too hasty. One will observe this pattern of responses without ambiguous belief aversion if equation (10) takes on the specific functional form

$$p_2 = (1/3)r_1 + (2/3)r_2. \tag{11}$$

Equation (11) is consistent with Bayesian learning for situations in which the respondent places a greater weight on the second study. This example highlights the care one must exercise in testing for the influence of ambiguous belief aversion.

If ambiguous belief aversion is of consequence, then we should rewrite the posterior belief equation (10) to take it into account. Since the role of risk ambiguity may be a nonlinear relationship, we will include both the linear and quadratic risk-range terms in the equation, that is,

$$p_2 = \gamma' p_0 + \xi_1' r_1 + \xi_2' r_2 + \Psi_1 R + \Psi_2 R^2. \tag{12}$$

Because the hypothesis is that evaluated at the mean risk level, the net effect of R is positive in the presence of ambiguous belief aversion (i.e., $\Psi_1 R + \Psi_2 R^2 > 0$), whereas ambiguous belief aversion has no role to play in a standard Bayesian learning model. If the influence of ambiguous belief aversion diminishes with the extent of the risk range, then $\Psi_1 > 0$ and $\Psi_2 < 0$, whereas an increasing incremental effect of ambiguous belief aversion will be indicated by $\Psi_1 > 0$ and $\Psi_2 > 0$.[16]

A final elaboration on the model is needed, since we do not observe p_0 and consequently cannot estimate the value of $\gamma_0' p_0$. Suppose, however, that the individual's prior and the precision of this prior is a function of his or her demographic characteristics, X_i. Taking the linear form, we have

$$\gamma_0' p_0 = \alpha + \sum_{i=1}^{a} \beta_i X_i, \tag{13}$$

then we can rewrite equation (10) as

$$p_2 = \alpha + \sum_{i=1}^{a} \beta_i X_i + \xi_1' r_1 + \xi_2' r_2 + \Psi_1 R + \Psi_2 R^2. \tag{14}$$

It is instructive to contrast this empirical test with earlier tests that have appeared in the literature. The Ellsberg urn model explicitly highlights the role of ambiguous probabilities, but may not be conclusive. One urn is uncertain, whereas another has properties known with precision. Some authors, included Ellsberg (1961), have speculated that respondents may believe the uncertain urn is being manipulated against them so that the stated probabilities will not be treated at face value. The underlying asymmetry in the experimental structure may capture more than risk ambiguity.

More recently, Kunreuther and Hogarth (1989) developed a test of an anchor and adjustment model that they contrast with a Bayesian expected utility model. Their innovative study strongly suggests the presence of ambiguous risk belief aversion but does not serve as a definite test of the Bayesian framework presented here.[17]

3. Empirical results

The sample used for this study consisted of 646 adults who participated in a survey regarding attitudes toward experimental risks. This sample was drawn at a shopping mall in Greensboro, North Carolina, where the demographic characteristics are broadly representative of the U.S. population.[18] The average education of the sample was 13.4 years; 49% of the sample had household income over $30,000, and the remainder had income below that amount; 57% of the sample were married, and 44% were males.[19]

Each subject was given a computer-administered questionnaire that elicited preferences with respect to moving to either area A or area B. This risk information provided to the subjects concerning area A varied, since four different risk pairs were communicated. These risk pairs, which were all in terms of the incidence of the disease per million population, were the following: (150,200), (110,240), (125,155), and (105,135). The presentation order and the temporal order of these studies were varied so that in all there were ten different distinct sets of information provided; these are all summarized in table 2. The mean risk conveyed for area A was 175×10^{-6} for 8 of the 10 studies. As the mean responses indicate, the equivalent risk in area B is often quite different, especially when the area A risk range is large. The highest mean risk value is 197.45 for (110,240), where the studies had an explicit temporal order. None of the respondents indicated an assessed risk of zero or one, so econometric problems arising from observations at a limit did not arise.[20]

Table 3 reports the results of different ordinary least squares estimates of alternative specifications of equation (13) above.[21] Since there was evidence of significant heteroskedasticity in the results, the bracketed values in table 3 present the heteroskedasticity-corrected standard errors based on the procedure developed by White (1980). These adjusted standard errors have similar implications with respect to the significance of the key coefficients of interest.

Table 2. Summary of experimental treatments

Risk level in studies of area A risk	Studies had temporal order	Equilibrating area B risk—mean (std. error of mean)
150,200	No	178.35 (1.24)
150,200	Yes	181.67 (1.10)
200,150	No	177.88 (2.67)
200,150	Yes	174.13 (1.18)
110,240	No	191.08 (3.95)
110,240	Yes	197.45 (2.95)
240,110	No	170.35 (5.78)
110,240	Yes	159.19 (3.84)
155,125	No	134.90 (1.07)
135,105	No	130.38 (0.39)

The units of the dependent variable in the equations in table 3 are the number of illnesses (per million population). To convert these estimates into a probability, one must consequently multiply the coefficients by 10^{-6}. The nonrisk variables are dummy variables (d.v.), except for Education (years of schooling).

Equation (1) in table 3 represents the basic version of the learning model in which the intercept captures the role of the prior probability beliefs, and the coefficients of r_1 and r_2 represent the informational weights placed on studies 1 and 2, respectively. The risk ambiguity aversion terms are not included. The intercept term in equation (1) is not statistically significant, which indicates that on average the influence of prior probability beliefs on risk perceptions is not significantly different from zero. This result does not necessarily imply that the prior risk assessments for these events are not significantly different from zero, although this may in fact be the case because these are very low probability events. Rather, the results suggest that the combined influence of the relative informational weight placed on the prior multiplied by the value of the prior is not significantly different from zero, or $\gamma' p = 0$. Alternative specifications of the model in which the intercept was permitted to vary with the health outcome also did not yield significant effects.[22] Since the magnitude of the prior assessed risks and the severity of the outcomes of the two health risks are likely to be similar, this result is not surprising.

The two information weights for the first and second risk studies are each significantly different from zero, since the weight ξ_1' on r_1 has a value of 0.41, and the value of ξ_2' for r_2 is 0.55. A somewhat greater weight is placed on the second study mentioned to respondents.

These results for equation (1) change very little once a series of demographic characteristics is added in equation (2). The values of the two informational weights for ξ_1' and ξ_2' drop to 0.39 and 0.52, but are not much affected by inclusion of the demographic variables. The only statistically significant (95% confidence level, one-tailed test) demographic variables in equation (2) are whether the respondent is employed (which has a negative effect on risk perceptions) and whether the respondent has an income level above $30,000 (which has a positive effect on risk perceptions). These and the other demographic variables are intended to capture differences in prior probability beliefs across different individuals.

Table 3. Estimates of the risk perception equation[a]

Independent variable	Coefficients (std. errors) [Heteroskedasticity-adjusted std. errors]					
	1	2	3	4	5	6
Intercept	12.074 (7.992) [4.206]	16.326 (12.847) [8.986]	10.601 (13.327) [9.009]	5.665 (12.156) [8.773]	11.389 (12.507) [8.508]	14.812 (12.402) [8.024]
RISK1(r_1)	0.412 (0.028) [0.023]	0.390 (0.033) [0.028]		0.558 (0.035) [0.034]	0.528 (0.039) [0.034]	0.195 (0.094) [0.042]
RISK1(r_1) × temporal order				−0.215 (0.020) [0.028]	−0.213 (0.020) [0.028]	−0.216 (0.020) [0.028]
RISK2(r_2)	0.547 (0.025) [0.016]	0.524 (0.032) [0.024]		0.401 (0.035) [0.034]	0.375 (0.038) [0.034]	0.041 (0.094) [0.040]
RISK2(r_2) × temporal order				0.219 (0.023) [0.031]	0.216 (0.023) [0.031]	0.213 (0.022) [0.031]
RISK1(r_1) + RISK2(r_2)			0.476 (0.032) [0.021]			
RISK RANGE (R)					0.043 (0.023) [0.025]	3.163 (0.806) [0.351]
(RISK RANGE)^2						−0.017 (0.004) [0.002]
Education (years)		0.339 (0.393) [0.335]	0.291 (0.406) [0.341]	0.337 (0.363) [0.325]	0.318 (0.362) [0.322]	0.423 (0.359) [0.322]
Employed (0–1 d.v.)		3.280 (2.221) [2.198]	3.383 (2.294) [2.307]	3.711 (2.049) [1.982]	3.729 (2.045) [1.988]	3.982 (2.024) [1.946]
High income (0–1 d.v.)		−3.575 (1.920) [1.920]	−3.883 (1.983) [1.997]	−4.734 (1.774) [1.839]	−4.794 (1.771) [1.829]	−4.252 (1.757) [1.821]
Lymph cancer knowledge (0–1 d.v.)		2.366 (2.795) [3.156]	2.079 (2.886) [3.228]	3.516 (2.582) [2.867]	3.439 (2.577) [2.838]	3.607 (2.549) [2.818]
Nerve disease knowledge (0–1 d.v.)		1.115 (3.732) [3.157]	0.767 (3.854) [3.357]	0.309 (3.444) [3.091]	0.663 (3.442) [3.056]	−0.038 (3.409) [3.081]
$\bar{R}^2$	.43	.43	.39	.51	.51	.53

[a]Other variables included in equations (2)–(6) are respondent sex, number of people in household, age, age variable missing dummy variable, life insurance coverage, and marital status.
Note: d.v. = dummy variable.

Equation (3) in table 3 constrains the coefficients ξ_1' and ξ_2' to be equal, since as it estimates the average information weight placed on the sum of the two risks implied by the studies. This estimate yields an average value of 0.48 as the informational weight on the studies. The appropriate F-test suggests that the coefficients ξ_1' and ξ_2' for the two risk variables are statistically different from one another.[23]

The findings of equations (1) and (2) in table 3 are consistent with a Bayesian learning model. One cannot distinguish at this juncture which particular model of learning receives the strongest support, since the empirical analysis for the first two equations in table 3 only reflects the most rudimentary aspects of learning. We can rule out, however, the model of alarmist learning, since there is no evidence that individuals respond excessively to the risk information presented, as these tests have been defined in table 1.

Equation (4) recognizes the likely influence of the temporal order of the studies on the informational weights ξ_1' and ξ_2' that the respondents place upon the risk studies. In particular, if one knows that the second study mentioned in the survey also was undertaken after the first study mentioned, then there is reason to believe that the second study extends the initial one or was based on more recent scientific methods and consequently should receive greater weight. A strong effect of this type is the fact observed. The first study presented to the respondents receives a relative informational weight of 0.56, but if a temporal order is indicated the weight is 0.22 less, for a net informational weight of 0.34. In the case of the second study mentioned, the weight placed on this study is 0.40, but if there is an explicit temporal order indicated the second study receives a weight of 0.62. As a consequence, in situations in which there is an explicit temporal order, the second study receives roughly double the weight as does the first, whereas in situations in which there is no temporal order indicated, respondents place a greater weight on the first of the two studies mentioned, perhaps because of its greater prominence.

These results are supportive of several models in table 1. One can rule out the naive Bayesian, since informational weights are not identical for the studies mentioned. There is evidence in support of the attentive Bayesian in the case of studies presented in which there is a temporal order, since the more recent studies receive the expected greater weight. However, for studies in which there is no temporal order, the respondents behave in a manner that is consistent with a Bayesian who has cognitive limitations. The first study mentioned in the interview will be more prominent for respondents who process the information hastily or incompletely, and consequently the first study receives greater weight. The overall impression conveyed by these results is one of the respondents who act in a manner one would expect given a rational learning process, with the only deviation from full rationality being that they pay more attention to the first study mentioned in the interview.

The final two equations in table 3 extend the model by including the ambiguous belief aversion terms in the analysis. In each case the linear risk range term is positive and statistically significant. For equation (5), the results imply that for each additional case per million in terms of the risk spread, the effect is to raise the risk perception by 0.043. Thus, in the case of the pair of risk studies posing risks per million of (110,240), which is the largest risk spread considered in the experiment, the role of risk ambiguity is to raise the assessed risks per million by 5.6 relative to the situation with no ambiguous beliefs.

The findings in equation (6) indicate that the role of risk ambiguity is positive, but that it diminishes with the extent of the risk spread. In particular, the impact of risk ambiguity displays a strong nonlinearity that indicates a diminishing role of ambiguity belief aversion as the extent of the risk spread is increased. For the largest risk range with risk pair (110,240), the findings in equation (6) imply that risk ambiguity raises the assessed risk by 123.89×10^{-6}. This effect is 77% of the mean risk of the two risk studies. The potential influence of ambiguous belief aversion is consequently substantial for the quadratic specification in equation (6).

For the specifications including risk ambiguity, individual employment status continues to exhibit a significant positive effect on risk perceptions, and being in a high income group has a negative effect. The lymph cancer knowledge variable falls short of a statistical significance at the 5% level, but not at the 10% level.

Other variations that may reflect variations in the role of risk ambiguity proved to be inconsequential. For example, interactions of the risk-range variable with the temporal-order variable did not yield a statistically significant effect.[24] This modification was of potential theoretical interest, since the temporal order influences the relative information weights ξ_1' and ξ_2'. The absence of a significant effect of the temporal-order interaction suggests that, given the experimental variations in informational content, only the risk range, not the precision, affects the estimates of the ambiguous belief aversion effect. Interactions of the ambiguous belief variables with respondent income also proved not to be statistically significant.[25]

4. Conclusion

Individual responses to ambiguous risk information suggest that the character of this response is more subtle than most previous treatments have suggested. The perfect learning model captured through a Bayesian framework received some support in that individuals weight the information that they received, and they place a greater weight on the scientific evidence that is more recent and that should be more credible. Moreover, these responses are not so excessive that they indicate alarmist learning responses.

The findings did, however, indicate two limitations on this learning process. First, individuals' cognitive limitations affect the manner in which they process risk information. The first study presented to them within a survey context has greater prominence and as a consequence receives greater emphasis in forming risk beliefs.

The second departure from the standard learning model pertains to the role of ambiguous beliefs. There is strong evidence of ambiguous belief aversion, even after one takes into account the full ramifications of a Bayesian learning process. This ambiguous belief aversion increases with the extent of the risk range, but at a diminishing rate. What is perhaps most important about this result is that the role of risk ambiguity is found to be significant within the context of an empirical test that utilizes a fully developed Bayesian expected utility model as the reference point for analysis.

These results do not distinguish whether the source of the ambiguous belief aversion stems from a misperception of the probabilities or from an omitted aspect of individual preferences. What is clear is that a Bayesian expected utility model cannot fully capture the influence of ambiguous risk beliefs. The precise probability that establishes indifference in the reference lottery is higher in situations in which one is facing a lottery involving ambiguous risk beliefs. This effect could arise if risk ambiguity entered negatively as a component of a multiattribute utility function or if it affected the perception of these probabilities in a manner that led to a violation of the standard probability axioms.

The findings do, however, suggest that the role of risk ambiguity is not limited to the narrow experimental context addressed in the Ellsberg paradox. In situations in which individuals acquire risk information from diverse and possibly conflicting sources, they will behave in many ways that are consistent with a standard Bayesian learning model. However, there is an additional component to the choice process involving ambiguous risks that cannot be reconciled within a Bayesian decision framework.

Notes

1. For a review of many of these anomalies, see Kahneman and Tversky (1979), Tversky and Kahneman (1986), Machina (1987), and Camerer and Kunreuther (1989a). Zeckhauser (1986) provides a perspective on the relation of these results to economic analysis.
2. In some economic models there may be a preference for risk ambiguity. The evidence presented in Viscusi (1979) and Viscusi and O'Connor (1984) indicates that in multiperiod job choice context, workers should have a preference for ambiguous risks. This result appears in Viscusi and O'Connor's study of worker responses to risk information, but it hinges critically on the role of learning in a multiperiod adaptive choice context. This article will focus on single lotteries so that such concerns will not enter.
3. Models along these lines include, among many others, Einhorn and Hogarth (1985) and Kunreuther and Hogarth (1989, 1990).
4. See Ellsberg (1961) and Viscusi (1989).
5. Smith (1969), Winkler (1991), and Heath and Tversky (1991) develop analyses based upon an effect of ambiguity on preferences.
6. Camerer and Kunreuther (1989b) also discuss these issues. The insurance industry is also heavily regulated and there are, for example, constraints imposed on reinsurance and related aspects of the rate-making process. Moreover, insurers are dealing with sequences of lotteries over time.
7. A copy of the text of the survey is available upon request from the authors. Additional background appears in Viscusi, Magat, and Huber (1990).
8. Alternatively, both probabilities may be affected, but altering only one probability simplifies the model.
9. One could also make ambiguity belief aversion a function of the payoffs, but as we will see below, the payoffs will affect the equilibrating value of s without this complication.
10. See Smith (1969), Heath and Tversky (1991), and Winkler (1991) for advocacy of this approach.
11. If we let the utility function in good health be given by $cU(Y)$, where c is a positive constant, one can show that $\partial s/\partial c < 0$ for both models above. The positive effect of increases in $V(Y)$ on s is determined analogously.
12. Their findings are also consistent with the specific models they advocate as well.
13. The survey did not elicit the prior assessed risk, as in Viscusi and O'Connor (1984), because the extensive information presented to the subjects about the diseases within the context of this survey would have altered their beliefs and stated perceptions.
14. The beta distribution can assume a wide variety of skewed and symmetric shapes and is ideally suited to analyzing Bernoulli-type processes such as this. It is, for example, more flexible than the normal distribution, which yields the same functional form for the posterior probability function given below. See Viscusi

(1979) and Viscusi and O'Connor (1984) for motivation of the particular parameterization of the beta distribution used here. Smith (1988) has also this formulation, but with reference to the normal distribution, which yields the same functional form for estimation.

15. See Viscusi (1989, 1990) for a Bayesian explanation of such behavior.
16. Although R can take on only three possible values—30, 50, and 130—the linear and quadratic terms impose structure on the nature of their influence in the risk perception equation.
17. First, they focus on the behavior of the median respondent rather than developing an explicit statistical analysis of the entire data set. Second, the test they suggest for determining the validity of the Bayesian model is suggestive but not conclusive.

    Kunreuther and Hogarth (1989) describe this test in section 3.4.1 (pp. 18–19) of their paper. Their study presents subjects with a single piece of risk information, leading to the formation of assessed probabilities $p_1(r_1)$ values that should follow equation (8) if subjects are Bayesian. They hypothesize that one should find that

    $$p(r_1) + p(1 - r_1) = 1$$

    if individuals learn in a Bayesian manner. This should be the case if the events with stated probabilities r_1 and $1 - r_1$ are complementary. However, if they are not complementary, there is no additivity requirement. If we implement equation (8), we find that the Bayesian updating requirement is somewhat different, or

    $$p(r_1) + p(1 - r_1) = (2\gamma p_0 + \xi_1)/(\gamma + \xi_1),$$

    which only equals 1 if $p_0 = .5$. The additivity hypothesis would be correct if there were a probability .35 of some event and a probability .65 of some complementary event. Kunreuther and Hogarth's study, however, described separate insurance risks that posed stated risks of .35 and .65, respectively. Respondents may bring to an insurance experiment their prior beliefs concerning the risk (Viscusi, 1989).
18. In several earlier studies we used a similar sample drawn from the same shopping mall. The Greensboro, North Carolina area is not the site of a major college and, because of its representativeness, is often used as a national test site for major consumer marketing efforts. Even if this were not the case, our main concern is with the character of individual behavior, not the specific magnitudes of the responses.
19. The average reported age of the sample members was 32, but 17% of the sample did not report the age value because of the sensitivity of this question for older respondents. A "missing" variable for the respondents without reported age values is included in the regression.
20. Thus, it is not necessary to utilize an estimation technique such as two-limit Tobit, which would be appropriate if there were a massing of observations at 0 and 1.
21. Ordinary least squares is utilized rather than logit or probit because the empirical predictions are tied to the linear form of the model. None of the predicted values of the equations were outside the range [0, 1], so recognition of the constraint on the estimated probabilities was not of consequence. In the case of equation (1) in table 3, one can verify by inspection that this condition is met.
22. Because of singularity problems, both an intercept and a dummy variable for the lymph cancer subsample cannot be included. If we omit the intercept, which is not statistically significant in any of the results in table 3, and include a dummy variable for the lymph cancer subsample, one obtains an estimated coefficient (std. error) of -2.119 (3.456) for an equation that parallels equation (6) in table 3. The differences in the character of the health outcomes and risk information across diseases (i.e., inclusion of the mean risk estimate in the information presented) do not lead to a significant shift in risk perceptions. Unfortunately, separate equations cannot be estimated for the diseases due to singularity problems.
23. More specifically, the calculated F statistic for the hypothesis that $\xi_1' = \xi_2'$ has a value of 43.2, which greatly exceeds the $F_{0.05}$ cutoff of 3.84 and the $F_{0.01}$ test cutoff of 6.63.
24. In particular, the parameter estimate was -0.010, with an associated standard error of .021. Because of singularity problems, temporal order could not be interacted with both RANGE and $(\text{RANGE})^2$.
25. The estimated coefficients and the standard errors obtained by adding these interactions to equation (6) in table 3 were 0.0403 (.4270) [0.1921] for RISK RANGE × INCOME and -0.0008 (0.0025) [0.0012] for $(\text{RISK RANGE})^2$ × INCOME. The calculated F statistic for the hypothesis that the RISK RANGE linear and quadratic terms are independent of income is 2.92, which is below the critical $F_{0.05}$ value of 3.00, so that the iterations are not jointly significant.

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